Nonuniform, Inhomogeneous, and Anisotropic Waveguides*

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Summary—**This paper discusses in a general way some of the properties of nonuniform, inhomogeneous, and anisotropic waveguides and transmission lines. No attempt is made to discuss in detail the behavior of any particular type of waveguide except for the purposes of example. Most of the points discussed are well known separately, but an attempt has been made to bring some of general properties of special types of waveguide together in order to unify them, and, thus, it is hoped, to contribute to the over-all understanding of guided wave phenomena.**

IN THIS paper I would like to discuss briefly some of the properties of *non*uniform, *in*homogeneous, and *an*isotropic waveguides and transmission lines. This is, of course, too broad an area to cover in detail, but I will outline some of the special qualities of these types of waveguides.

The usual waveguide—which is uniform, homogeneous, and isotropic—has been commonplace in microwave applications for about two decades. The theory of such guide in dominant mode operation can be considered essentially complete, at least in its fundamentals. Important new work is being done today, however, on multimode propagation in uniform, homogeneous, isotropic waveguides.

It is my purpose to discuss here some of the *generalizations* of the usual waveguide structure. Recent researches have uncovered many interesting and useful properties of these generalized structures. They are generally referred to as *non*uniform waveguides, *in*homogeneous waveguides, and *an*isotropic waveguides. For the sake of clarity, the meaning of these terms is illustrated in Fig. 1.

By the term *nonuniform waveguide* is meant a waveguide whose characteristics change in the direction of wave propagation. Ordinarily the basic structure remains the same, but the proportions of the guide may change. The continuous impedance matching taper shown at the top of the figure is such a structure. The change of dimension may also occur discontinuously as shown at the top right, and thus, the familiar impedance matching transformer is also an example of a nonuniform waveguide. Another example, not shown in the figure, is the tapered load commonly used in microwave practice.

An *inhomogeneous waveguide* is one whose properties remain constant in the direction of propagation, but they may vary across the cross section of the waveguide as shown in Fig. 1 (b). Here again the change may occur either continuously or discontinuously.

In an *anisotropic waveguide* the medium properties are constant across the guide cross section *and* along the direction of wave propagation. In this type of medium, however, the magnetic induction vector and the magnetic intensity vector are not necessarily colinear. But they still remain proportional. That is, if the magnetic induction is doubled in magnitude, so is the magnetic intensity. This situation is completely described by considering the magnetic permeability to be a tensor rather than a scalar. Such a condition exists in a waveguide filled with saturated ferrite, for example. Anisotropies may be present in the electrical properties of the medium, but this is not nearly so common in current microwave practice.

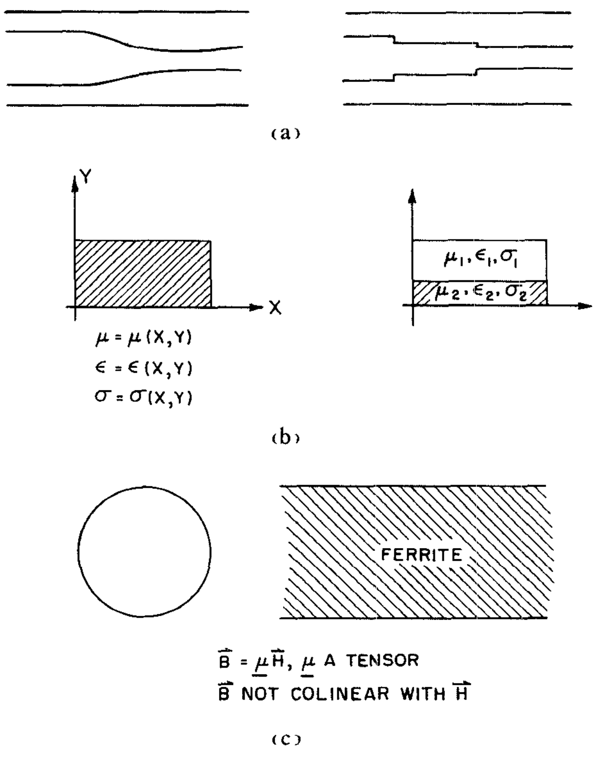

Fig. 1—Various waveguide types. (a) Nonuniform waveguides. (b) Inhomogeneous waveguide. (c) Anisotropic waveguide.

We ordinarily deal with various combinations of these waveguide types. An extreme example, perhaps, is a ferrite cylinder of finite length and tapered at the ends inserted into a larger circular waveguide. This configuration could be described as a *nonuniform, inhomogeneous*, and *anisotropic* waveguide.

*In*homogeneities and *non*uniformities are introduced into waveguides and transmission lines for *two* major

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reasons. Fig. 2 shows a number of transmission lines in which the desired behavior of the line is that of a uniform, homogeneous line. The deviations, in this case, are made for practical reasons such as the mechanical support of the conductors and suitability of the structure for printed circuit techniques. Here then, the special properties due to the *non*uniformities and *in*homogeneities are necessary evils, and are to be minimized as much as possible by suitable design. This situation is more characteristic of transmission lines than *uni*conductor waveguides. This is because of the multiplicity of conductors to be maintained in proper relation to each other in the case of transmission lines.

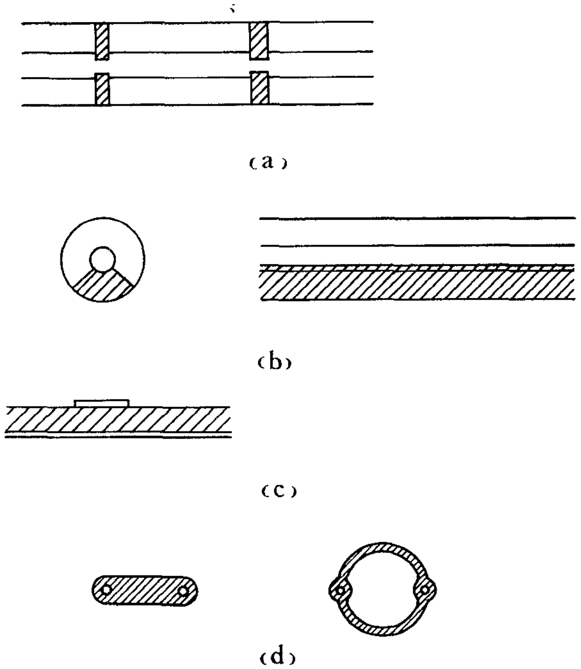

Fig. 2—Examples of composite and nonuniform transmission lines intended to simulate, in performance, homogeneous transmission lines. (a) Bead supported coaxial line. (b) Wedge supported coaxial line. (c) Strip lines. (d) Twin lead.

A more interesting situation is that in which *in*homogeneities and *non*uniformities are introduced in order to obtain properties which are not obtainable with uniform, homogeneous waveguides. Fig. 3 shows a number of such applications. The surface wave transmission line would not function at all in the absence of of the *in*homogeneity. Likewise, the nonreciprocal character of the Faraday rotator, shown in Fig. 3 (b), depends upon the *ani*sotropy of the ferrite material. For proper functioning of the elementary form of the linear accelerator shown, waves of slow phase velocity must be propagated in an empty space. This property depends on the inhomogeneity of the cross section.

A fourth application of considerable interest is shown at the bottom of Fig. 3. The structure consists of a rectangular waveguide with symmetrical dielectric slabs on each side. The investigation in which this structure was used dealt with the microwave resonances of the ammonia gas molecule. It was desired to expose a sample of ammonia gas to an essentially constant electric field at microwave frequencies, in order to determine the absorption spectrum. The unique thing about the structure shown is that in the dominant TE mode, proper proportioning of the dielectric constant or dielectric dimensions leads to a mode which has this property. The electric field in the empty space does not depend then on the cross-sectional coordinates, but is constant. Moreover, this condition is readily obtainable with low-loss dielectrics commonly used in microwave practice.

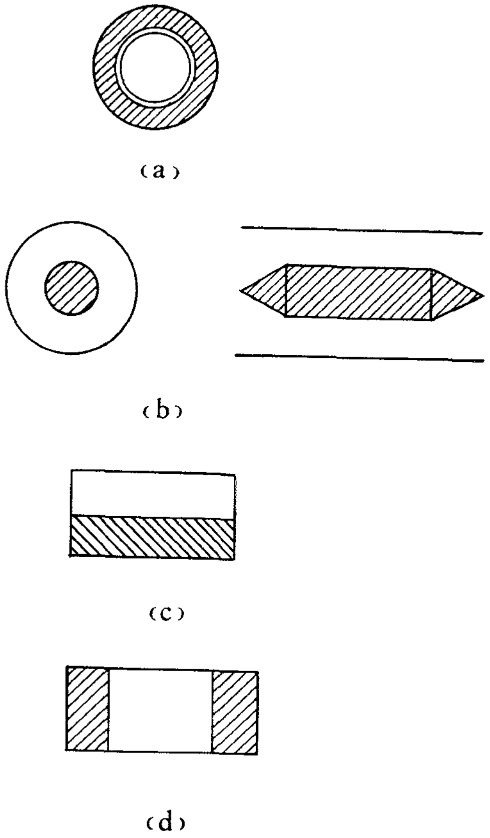

Fig. 3—Examples of inhomogeneous and anisotropic transmission systems in which the inhomogeneity and anisotropy is essential for the desired transmission properties. (a) Surface wave transmission line. (b) Nonreciprocal attenuator in circular waveguide. (c) Linear accelerator. (d) Composite rectangular waveguide for the investigation of microwave resonances of NH_3 molecules (E_y constant in open space).

The early theoretical investigations of *in*homogeneous waveguides dealt with particular types and their properties. In this manner, a number of unusual features of the mode structure of such guides were discovered. A general investigation of *in*homogeneous waveguides was undertaken by Professor Adler of M.I.T. as a topic for a doctoral dissertation. This was published in 1949. He found, among other things, that the free modes in an *in*homogeneous waveguide were *almost completely* of the hybrid type—that is, neither transverse electric nor transverse magnetic, but having axial components of both electric and magnetic fields. Professor Marcuvitz has more recently extended this work and shown that the set of free modes on a *closed in-*

homogeneous waveguide is complete—that is, that they are adequate to express the total field due to an arbitrary source distribution. This is not true, however, of *open inhomogeneous* guides, such as the dielectric rod waveguide. Here, an additional continuum of radiating modes is required for completeness.

Fig. 4 contrasts some of the properties of *in*homogeneous waveguides and transmission lines with those of homogeneous guides.

Homogeneous Waveguide	Inhomogeneous Waveguide
1) Modes either TE or TM. Low frequency mode TE and TM (TEM).	1) Modes hybrid, in general. Some or, all, may be TE or TM, but low frequency mode never TEM ($\sqrt{\mu\epsilon}\neq$constant).
2) Low frequency mode has static field distribution. $Z_0 = \sqrt{L/C}$, $V_P = \sqrt{LC}$.	2) Low frequency mode does not have static field distribution. Z_0 and V_p depend on frequency.
3) Guide wavelength directly from spectrum of cutoff frequencies. $\lambda_g = \dfrac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}}$	3) Guide wavelength found by numerical solution of transcendental equation, in general.

Fig. 4—Comparison of properties of homogeneous and inhomogeneous waveguide.

In the homogeneous guide the mode types are separable into two classes—TE and TM. When a low frequency mode exists, it is both TE and TM, that is, TEM. By way of contrast, in an *in*homogeneous guide the mode types are in general neither TE nor TM but hybrid in character. *Some* of the modes *may* be TE or TM, but the low frequency mode will never be TEM unless the free space propagation velocity is constant over the cross section.

For a homogeneous guide, the field distribution for the low frequency mode is that associated with the static charge distribution for the electric field and the static current distribution for the magnetic field. This field distribution does not depend on frequency, nor does the characteristic impedance and phase velocity which are expressible in terms of the static inductance and capacity. In an *in*homogeneous guide, on the other hand, the low frequency mode does not have the static field distribution except in the zero frequency limit. The characteristic impedance and phase velocity vary with frequency and only approach the static values in the zero frequency limit.

Another feature which leads to complications when one is dealing with an *in*homogeneous waveguide or transmission line is the determination of the guide wavelength, or phase velocity, for any particular mode. In a homogeneous waveguide, the guide wavelength is immediately obtainable for each mode, at any operating frequency, by means of the familiar relation shown on the figure. This situation no longer prevails with an inhomogeneous guide and the determination of guide wavelength normally requires the solution of a rather involved transcendental equation by numerical methods. The extent of the difficulties, naturally, depends on the merits of the individual problem.

These remarks may be illustrated by considering the simple example shown in Fig. 5.

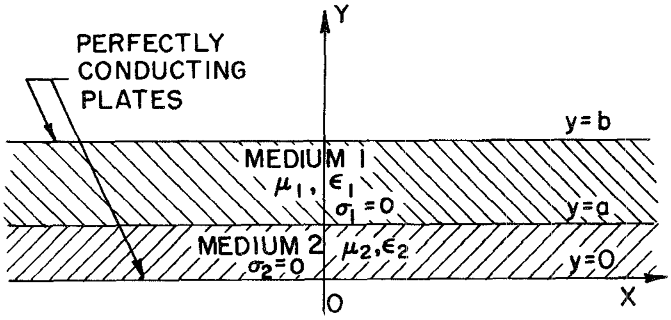

Fig. 5—Elementary type of inhomogeneous transmission line.

This is probably the simplest type of *in*homogeneous waveguide that one can think of. A parallel plate transmission line is modified by inserting a dielectric slab along one side. It is a useful example because the analysis is comparatively quite easy. There are a number of ways to attack these problems, but, in this case, the straightforward method of expressing solutions to Maxwell's equations in the two media separately, and matching boundary conditions is direct and convenient.

Attention will be focused here on the properties of this line as a low frequency transmission line. One of the primary quantities of interest in such a line is the cutoff frequency of the first higher order mode.

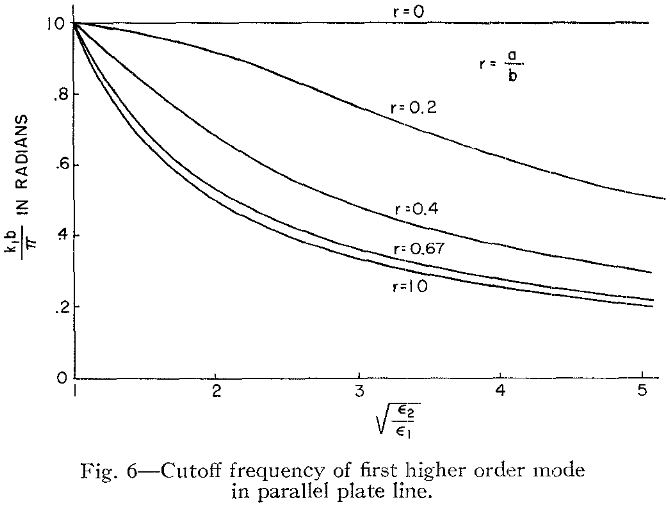

Fig. 6—Cutoff frequency of first higher order mode in parallel plate line.

Fig. 6 shows the cutoff frequency as a function of the relative dielectric constants of the two media for various amounts of dielectric. The line $r=0$, and the curve $r=1.0$, represent the limiting conditions in which the guide is entirely filled with one dielectric or the other. There are two features of these curves of particular interest. Firstly, the cutoff frequency is always between the limiting values for one dielectric or the other. Another interesting thing to notice about these curves is that the cutoff frequency of the first higher order mode decreases much more rapidly than in proportion to the amount of dielectric inserted. When the guide is two-thirds full, the cutoff frequency is nearly as low as when the guide is completely filled.

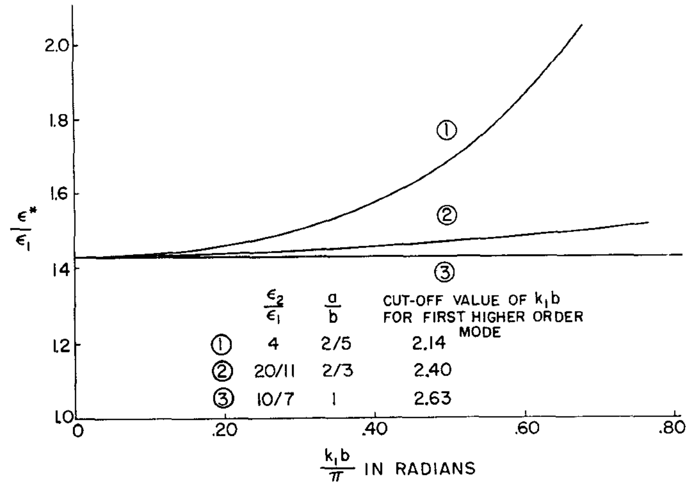

Fig. 7—Effective dielectric constant of parallel plate line.

Fig. 7 shows the "effective" dielectric constant of the guide for various proportions of dielectric. This is the dielectric constant from which the velocity, or guide wavelength, may be calculated. In the curves shown, the proportions are chosen so that the static "effective" dielectric constants are equal. The abscissa represents increasing frequency while the ordinate is the "effective" dielectric constant. These curves differ significantly from each other, and they increase significantly with frequency; except for the third case which represents a guide completely filled with one dielectric. The curves terminate at the cutoff frequency of the first higher order mode, which is the practical operating limit of the line in its low frequency mode.

Finally, Fig. 8 shows the variation of the field components between the plates for the same line proportions as used in the previous figure. The low frequency mode for this line is a transverse magnetic mode—not a TEM mode. The graph at the upper left shows the variation of the axial component of electric field. This can be compared to the transverse component, which is plotted to the same scale, in the curves at lower left. The transverse component of the magnetic field is shown at the lower right. A considerable variation of the field components is evident at this operating frequency which is near the cutoff frequency for the first higher order mode.

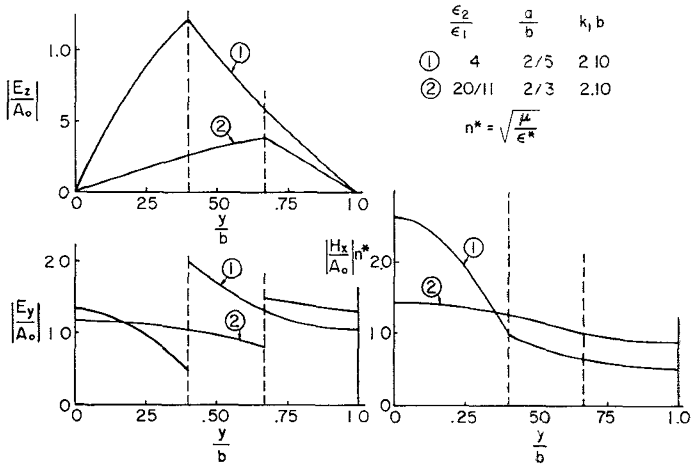

Fig. 8—Field components for parallel plate line.

The behavior of this prototype *in*homogeneous waveguide is qualitatively typical of all such guides. When a low frequency mode exists, it will not in general be transverse electromagnetic. It may be transverse electric, transverse magnetic, or hybrid. The characteristic impedance and phase velocity will depend on frequency to a greater or lesser extent depending upon the individual configuration. The cutoff frequencies will ordinarily lie between those of the corresponding homogeneous guide filled with dielectric of the maximum and minimum dielectric constants, respectively.

In summary, I would like to observe that we meet *non*uniformities, *in*homogeneities, and even *ani*sotropies almost continually in microwave transmission problems. Sometimes these occur as unavoidable necessities, and we minimize their effects as far as possible by suitable electrical design. In other cases, their effects are the very thing we need for the proper functioning of new devices. Their further study will undoubtedly throw new light on our general understanding of microwave transmission phenomena.